ACCESSIBILITY OF APPLICATIONS TASKS

Gloria Stillman
James Cook University
<Gloria.Stillman@jcu.edu.au>

This study identified the conditions that impeded or facilitated students' access to applications tasks at the senior secondary level. Impeding conditions comprised language related, representational, memory related, organisational and task specific conditions. Facilitating conditions comprised memory related, perceptual and engagement conditions. Some conditions were the result of the complexity inherent within the task whilst others arose from students' resource levels and as such acted as intervening conditions between task complexity and the difficulty the students experienced in accessing the tasks.

APPLICATIONS TASKS IN LEARNING AND ASSESSMENT

In Mathematics B, the main pre-tertiary mathematics subject in Queensland, the use of applications is advocated in order to develop positive attitudes towards mathematics and to increase mathematical competence by being concerned with "the modes of thinking which provide ways of modelling situations in order to explore, describe or control our social, biological and physical environment" (QBSSSS, 1992, p. 1). These reasons for advocating the use of applications in classrooms are based on widely held, though not universal (Verstappen, 1994), beliefs about the motivational powers of contexts and their use enhancing the links between the abstractness of school mathematics and the use of mathematics in the real world (NCTM, 1991; Niss, 1987). The use of contexts, while possibly addressing these concerns, presents other dilemmas, however.

The location of mathematical tasks in meaningful contexts for both teaching and assessment purposes is considered by many authors to be problematic (e.g., Amit, 1994; Patronis, 1997). As Boaler (1993), Clarke and Helme (1993) and Noss and Hoyles (1996) point out, contexts are not immutable and interpreted in some universal fashion by all students. Furthermore, Clarke and Helme (1993) suggest that even though a student may find a particular context interesting and engaging the demands of language and contextual detail may constitute an excessive cognitive load impeding progress towards a solution. Stillman and Galbraith (1998) infer from their study of applications tasks, that the focus of applications teaching should be on reducing the time students spend on orientation activities by "developing cognitive skills that facilitate more effective problem representation and analysis, and by promoting the development of metacognitive strategy knowledge" (p. 185) to facilitate appropriate decision making during orientation. It is imperative then that the conditions affecting students' access to applications tasks and the strategies they employ to take advantage of facilitating conditions or successfully overcome impeding conditions be identified. These strategies will be the subject of a future paper.

THE STUDY

This paper reports part of a much larger study which investigated the effects of context on students' approaches to, and performance on, applications tasks in Mathematics B. Specifically, this part of the study addressed the following research question: What conditions facilitate or impede students' access to applications tasks? These conditions may be (a) inherent in the nature of the task or (b) conditioned by the local context.

The study was conducted in two public high schools in a large provincial city. Data collection occurred over a six month period although contact with both schools was continuous during this time. Forty-three students in the last two years of high school (Years 11 and 12) participated in the study.

METHOD

Administration of the Tasks

All students were videotaped as they attempted to solve applications tasks individually with each student completing up to four tasks. An example is included in an appendix to this paper. These tasks differed in terms of familiarity, complexity and degree of contextualisation. The term *degree of contextualisation* is meant to convey the range of embeddedness that exists between the mathematics that can be used to model a situation and the description of the situation. This can range from the simple case where there is virtually no integration of the model within the context, the context merely acting as a border surrounding the mathematics which can be readily removed without loss of meaning, to the situation where the two are totally integrated and separation becomes difficult as the mathematics derives its meaning from the context. Although the majority of the tasks used in the study were closed rather than open-ended, the nature of some tasks provided scope for students to move in a direction appropriate to their perception of the problem. This occurred when tasks were more like true modelling tasks than mere applications or a fair degree of leeway was allowed in students' interpretation of the task.

Student Interviews

Sixty-four semi-structured stimulated recall interviews were conducted and recorded immediately following completion of the tasks. The videotape of their task solving sessions were reviewed by students in conjunction with the script of the task during the interview. Students were asked to draw diagrams, where appropriate, to illustrate their understanding of the problem context and what it was they had to do. The use of the videotaped task solving sessions as a visual stimulus throughout the interviews allowed both the interviewer and the interviewee to track the student's developing understanding of the task context by discussing their changing perception of the task as observed in their changing use of diagrams throughout the review of the task solving session.

Analysis

All interviews and task solving sessions were transcribed. The interviews were analysed using the qualitative data analysis software QSR NUD.IST (Qualitative Solutions and Research, 1997). NUD.IST facilitates "grounded theory" construction (Tesch, 1990) which attempts to capture and interrogate the meanings emerging from data. This is achieved by constructing and exploring new categories and themes as they arise from the data then refining these through a "process of progressive category clarification and definition" (Tesch, 1990, p. 86). This was done using a variety of matrix displays (Miles & Huberman, 1994).

RESULTS

Conditions Impeding Task Accessibility

Thirteen conditions were confirmed as impeding task accessibility at both sites. These were clustered into five groups in descending frequency of appearance: (a) language related conditions, (b) representational conditions, (c) memory related impeding conditions, (d) organisational conditions and (e) task specific conditions. These will now be examined in more detail.

Language Related Impeding Conditions

Language related conditions accounted for 62 of the instances of conditions impeding task accessibility. Language problems that stemmed from the use of technical language to describe the context, comprehension difficulties and unusually worded tasks were reported in 37 of the 64 cases of the study. This was by far the most frequently reported problematic condition. In addition, cuing words in the problem statement that were of either a mathematical or contextual nature were reported as not being salient for the task solver in 19 cases. Lengthy problem statements were also seen as a problem in seven cases.

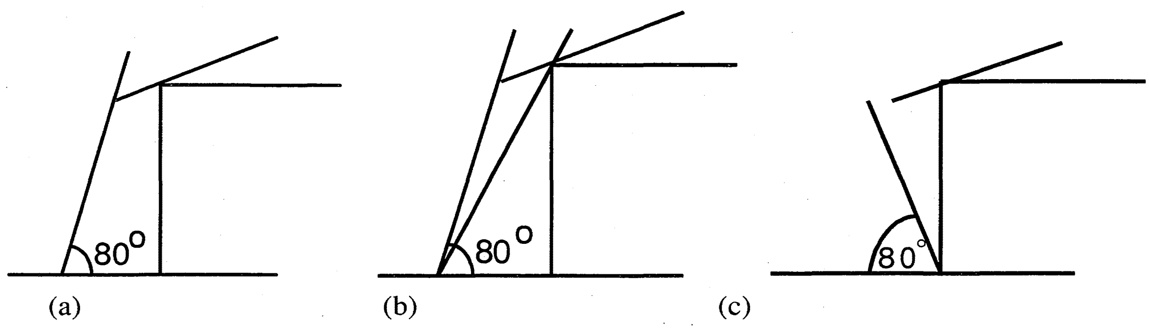
The nature of the context in which a task is set may entail the use of technical vocabulary associated with the context. Ideally, the task will be worded in such a way as to minimise the use of such vocabulary and to explain where necessary. However, even then some students will have difficulty. However, even normal everyday language or the language typically associated with particular mathematical contexts can cause difficulties which erode students' confidence in their ability to tackle the task at hand. In addition, students become very used to the wording of tasks from textbooks and the classroom. If the wording is not what is expected, this will just be a distraction for some students, whilst for others it can present an insurmountable difficulty.

Mathematical words and terms are often used in textual descriptions of tasks to clarify particular mathematical features of the task but these can impede rather than facilitate task access when their salience for the student is so low that they are ignored as is shown in the following exchange. The task statement defined the pitch of the roof as "the angle the slope of the roof makes with the *horizontal*" [italics added] but the student labelled this angle as being between the roof and vertical wall in her diagram.

> I: What are those two things?
> B3(TASK 18): That's the roof and the vertical line.
> I: Is that what it says?
> B3(TASK 18): The angle of the slope of the roof with the horizontal. The pitch of the roof is 12 degrees. So, like...this is like 12 degrees (draws over line of roof).
> I: And where would the horizontal be?
> B3(TASK 18): Here (draws in horizontal line marking top of wall).
> I: Yes, so if it is there, where would the 12 degrees be?
> B3(TASK 18): In here (marks in 12 degrees between roof line and horizontal).

Everyday language used to describe the context can also involve critical cues which students do not notice until their salience is raised as is shown by this further exchange. The student rereads the task statement and then attempts to mark in the angle of elevation of the sun (see Figure 1a). In this case the observer would be out from the building not at the base of the wall as implied by the fact that the wall is "*just* in shade" [italics added].

Figure 1
First (a) and Final (b) Version of B3's Third Diagram and her Fourth Diagram (c).



> I: Umm, that is one possibility, isn't it? Now what else does it tell you, you have to have? ... [Student rereads problem.] So where is your shade going to be?
> B3(TASK 18): Awh, umm, in here, this area here, down the bottom (indicating the ground beside the shed).
> I: Did it say that? If you were talking about that how would you say that? Would you be talking about the wall?

B3(TASK 18): The walls of the shed are in shade.
I: Yes.
B3(TASK 18): So, this is in the shade. (Draws down line representing external wall.)
I: Uhuh, but it has got one word...it is very critical if you have a look at the word about the walls in shade.
B3(TASK 18): The external walls...from the roof...from the roof overhang are just in the shade from the roof overhang.
I: What does the word 'just' mean here?
B3(TASK 18): Just sort of really close.
I: The wall is just in shade.
B3(TASK 18): Awh, so like, *I thought it meant that it was in the shade from the...I thought the overhang was in the shade as well.* So it's not that...*It's the wall is just in the shade* so it is like this.

At this point the student not only has to realise the significance of the word "just" but also she is now forced to be more flexible in her thinking and look at the situation from another angle. As she has focussed her attention on one particular visual representation for so long it takes her quite an effort to break her train of thought, being happy to accept an obvious incongruency in the situation before being forced to make the effort to think of other possibilities (See Figures 1b and 1c).

Seven students commented on the length of some of the tasks as being "off-putting". However, all but two of these students were able to overcome any difficulties this may have caused. They appeared to be speaking on behalf of others (that is, how they perceived other students without their skills might react). However, not all students, whom these students would have expected to have this reaction, agreed. Student A90 was one of the students in the study achieving at the Very Limited Achievement level in her school program. She felt the length of the Ecosystem Problem (one A4 page) was an advantage as it was "very descriptive".

Representational Conditions

Representational conditions accounted for 40 of the reported instances of conditions impeding task accessibility. These consisted of difficulties mathematising the context (18), integrating given or derived contextual information (12) and difficulties extracting the mathematical information from the context (10).

At times, students will understand the context fully and know what it is that they want to do but they are unable to do this as they can not mathematise the situation. Often this was demonstrated by their inability to draw a diagram representing the problem situation despite readily acknowledging the need to produce a visual representation. At other times, it was the extraction of the mathematical details from the context that was the problem.

Sometimes, students did not perceive in a situation that which was quite obvious to others because they were unable to integrate cues. They had a tendency to focus attention on parts of the situation rather than the whole. In the Road Construction Problem, B4 was so focussed on the angles made by the slope of the bank that it didn't occur to her that they formed parts of triangles so in her diagram she failed to join up the lines representing the sloping bank with the top of the bank. Another reason students had difficulty in this task with formulating a representation of the problem situation was that they were unable to integrate all the separate pieces of information from the context with their encyclopaedic knowledge of the world. Several students mentioned how they had to realise the dirt had to be removed to form the new lane before they could mentally integrate all the requirements of the task. In a richly elaborated context the task of integration becomes overwhelming for some students.

Memory Related Impeding Conditions

Memory related impeding conditions were problematic in 32 instances. These resulted from recall difficulties (15), the recall of metacognitive person or task knowledge that discouraged a student from accessing the task (8), the fact that the mathematics involved in particular tasks was not studied recently (6) and interference from recalled prior knowledge of the situational context (3).

Recall difficulties mainly involved the recall of a procedural skill that the student felt would assist in solving the task. There were examples, however, where the problem was the recall of a formula or concept. On two occasions prior knowledge which would have facilitated student's accessing and solving the task at hand was not recalled.

Recall of metacognitive knowledge and beliefs about themselves as task solvers can affect the outcome of students' cognitive activities by discouraging them from accessing a task because they believe it will be difficult for them or their chance of success is low. Similarly, beliefs or preferences for particular types of tasks can affect whether or not a student will access a task. Student A18's initial reaction on seeing the Shaft Problem was to think that it would be difficult to solve because it may involve a derivative and he did not consider himself to be "very good at derivative questions". These beliefs or preferences have developed from students' past experiences with similar tasks.

Recency of study of the mathematics involved appears to be a two-edged sword. If students have consolidated the skill or knowledge it is a simple matter of recall with the more recent the topic has been studied the more likely it will be readily recalled. Students at both sites suggested that by about a month without revision the memory trace is beginning to fade whilst longer periods certainly lead to forgetting. If, however, the skill or knowledge has not had time to be consolidated because it has been studied too recently students will still have difficulty applying it. Attempts were made to avoid this occurring in this study by pre-testing students for mathematical skills and then using tasks where they had demonstrated facility with the skills involved. Even then, there were isolated cases where students who had demonstrated that they could use the required skill on the pre-test were unable to recall it during the task-solving session three days later.

There were three cases where student's prior knowledge interfered with their development of a solution, however, in two of these cases the students involved were able to overcome this. In the Tide Problem, for example, student B7's belief that tides repeated every 12 hours interfered with the data given in the problem but she was able to recall the problem conditions and correct her error.

Organisational Conditions

Organisational conditions consisted of difficulties formulating a plan of attack (15) or establishing a goal (9). Inability to formulate a plan of attack was instrumental in several students' failure to access the task at hand as they did not know where to begin. Both comprehension difficulties which are purely language related and contextual demand can lead to a student not being able to decide what it is they are expected to do in a task or students can misinterpret what it is they are expected to find. Student A61 struggled to establish any meaning in the context of the Road Construction Problem where she was expected to check the angle of the bank beside the road. Instead, she misinterpreted her goal as finding the best angle at which to have the road inclined.

Task Specific Conditions

Only one of the task specific conditions was confirmed at both sites. It was the situation where the mathematics required was not obvious in a task which was reported as problematic in six cases. In the Ecosystem Problem and the Road Accident Problem, for example, the mathematics involved was considered by some students to be rather elusive as several options occurred to them but none of these stood out from the others.

Conditions Facilitating Task Accessibility

Conditions facilitating task accessibility which occurred at both sites were grouped into three clusters in descending order of frequency of appearance: (a) memory related conditions; (b) perceptual conditions, and (c) engagement conditions.

Memory Related Facilitating Conditions

Memory related conditions were reported as facilitating task accessibility in 45 instances. The majority of these were almost evenly divided between the recall of a similar or parallel task (19) and the recall of prior knowledge (18). The fact that the mathematics involved was well rehearsed was reported as making the task more accessible in six cases. Metacognitive person or strategy knowledge encouraged engagement with the task in two cases, one from either site.

Some students were able to make use of the similarity between the task they were attempting to solve and previous tasks they had solved. These students capitalised on the transfer between mathematically similar problems even though the contexts were different to facilitate their access to the problems in the study.

Relevant prior knowledge can reduce both the contextual and mathematical demands of the task. Relevant declarative or procedural knowledge from other subject areas such as Physics can often be of use also. The recall of relevant episodic or encyclopaedic knowledge facilitated the student's understanding of the task (Stillman, 1998). The prior experience of having seen a particular mathematical model used to represent a physical phenomenon was sufficient to facilitate access to a less constrained applications task such as the Tide Problem.

Although information stored in Long Term Memory does not decay, it does become more difficult to retrieve if it is not too frequently recalled (Best, 1995). Information which is well rehearsed is much more easily retrieved. Consequently, when students were set tasks that involved well rehearsed mathematics they found this helped them access the task.

As was indicated in the impeding conditions, past experience can have an effect on students' personal metacognitive knowledge. This is not always negative. Repeated success with particular strategies boosts a student's confidence in taking risks and using these strategies on future tasks.

Perceptual Conditions

Perceptual conditions facilitating task accessibility consisted of the recognition of trigger words and visual features in the task statement or any visual representation accompanying the task statement in 14 cases, being able to visualise the situation so as to understand it fully in 10 cases, and the instantaneous recognition of mathematical cues in three cases.

Readily identifiable task characteristics which include specialist vocabulary such as "incline", "angle of elevation", "rate of growth" or "slope" and typical diagrams or graphs make tasks more accessible to those who recognise them and realise their significance.

Several students believed that it was necessary for them to visualise the situation described in a task before they could solve it. Students who used this technique successfully spoke of acquiring it from teachers who taught them in the Junior secondary school. When this technique was used, the point of understanding when all the pieces fell into place was quite sudden and observable on the videotapes of the task-solving sessions.

Sometimes the recognition of mathematical cues in a context is instantaneous and the person doing the task is unable to articulate why or how they recognised these so quickly. They are unaware of even having thought about it. It is possible that the mathematical cues are just so explicit they are instantly extractable or, alternatively, that such thinking is unavailable for conscious introspection.

Engagement Condition

There was just the one engagement condition confirmed at both sites. In 14 cases, engagement with the task context was seen as facilitating a student's access to the task. This was evidenced by students being able to integrate various combinations of the following: (a) contextual information from the task statement, (b) prior knowledge of the situational context, (c) mathematical information given in the task statement or (d) mathematical information derived from information given in the task statement.

DISCUSSION

Some of these conditions which facilitate or impede students' access to tasks are the result of the inherent complexity of the task whilst others arise from students' resource levels. When the conditions facilitating task access are examined it is apparent that some conditions appear to reduce the difficulty level of a task for particular students whilst others contribute to reducing the complexity of the task. The conditions that reduce difficulty are either personal in nature (e.g., being able to visualise or possessing prior knowledge of the situational context) or attributes of the task that are susceptible to individual variation when a particular student interacts with the task (e.g., how recent a particular piece of mathematics required in the task has been studied). On the other hand, the complexity of the task appears to be reduced by particular attributes of the task (e.g., the presence of trigger words or visual features). An examination of the impeding conditions confirms this separation of task difficulty from complexity on the basis of task difficulty resulting from students' personal attributes and their interaction with the task. Students' resource levels thus act as intervening conditions between the complexity of a task and the difficulty level the students experience in attempting to access the task. These intervening conditions explain the different consequences that occur (e.g., different difficulty levels or whether impeding conditions are overcome) when different students attempt tasks of the same complexity.

CONCLUSION

This paper sought to identify the conditions affecting students' access to applications tasks as contexts can facilitate access for some, but not all, students. By documenting conditions that facilitate or impede task access and the strategies that students use to take advantage of facilitating conditions or successfully overcoming impeding conditions, teachers are provided with the means to develop teaching activities which focus on reducing the time students spend on orientation activities during the solution of these tasks. Such activities should promote the development of cognitive skills that ensure more effective problem representation and analysis and metacognitive strategy knowledge which facilitates appropriate decision making during orientation.

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APPENDIX

The Shed Problem (Task 18.1): An owner-builder is designing a shed for her backyard in an area that is affected by cyclonic winds. The length of the overhang of the roof is to be such that in mid-summer, when the angle of elevation of the sun is 75°, the 2 metre external vertical walls of the shed are just in shade from the roof overhang. She wishes to use Trimdeck roof sheeting. Because of the cyclonic wind classification for the site, council regulations restrict the overhang width (the horizontal distance from the outside edge of the roof to the wall) for the particular design to a maximum of 520 mm. If the pitch of the roof (the angle the slope of the roof makes with the horizontal) is 12°, is it possible for the owner-builder to meet council regulations and keep the walls shaded as stated above?